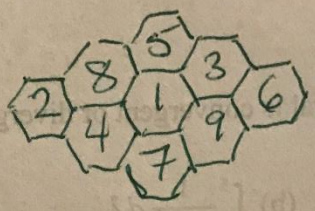
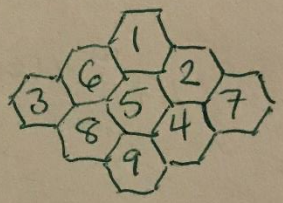
Puzzle Contest Solutions

1. There are 30 cubes. If you paint one side of a cube one color, you can turn it any way and get that color into any position. Then there are 5 choices for the color opposite the first. The third color can go into any of the remaining four positions without making different cubes. There are 3 choices for where to paint the fourth color, then 2 choices for the fifth color, and 1 for the sixth color, making the number 1*5*1*3*2*1=30.
2. The maximum size set will have 57 cards. The pattern is that for *n* items on a card, you need to have n^2-n+1 items and n^2-n+1 cards. Start with smaller numbers. Look at 2 items per card, then 3, then 4. Find the pattern.
3. The party is at house #84 on a street with 119 houses. Let *p* be the party house number and *h* be the total number of houses. Then $p^2+p=\frac{h^2+h}{2}$, so $h=\frac{-1+\sqrt{8p^2+8p+1}}{2}$. Party house 14 on a street of 20 would work, but the problem states there are more than 20 houses.
4. The man is 48 and his children are 2, 5, 8, 11, 14, 17, 20, 23, and 26 or the man is 96 and his children are 4, 10, 16, 22, 28, 34, 40, 46, and 52. Let *y* be the age of the youngest child, *d* be the difference between consecutive children, and let *m* be the man's age. Then $m^2=9y^2+72dy+204d^2$.
5. Mary's husband shook 4 hands. The only way the other nine people could all shake hands a different number of times is if spouse 1 of couple 1 shook 8 hands while spouse 2 of couple 1 shook none, spouse 1 of couple 2 shook 7 hands while spouse 2 of couple 2 shook 1hand, spouse 1 of couple 3 shook 6 hands while spouse 2 of couple 3 shook 2 hands, and spouse 1 of couple 4 shook 5 hands while spouse 1 of couple 4 shook 5 hands while spouse 2 of couple 4 shook 3 hands. This leave 4 and 4 for Mary and her spouse.
6. It takes 15 line segments. A 5x8 grid can be made with 6 vertical and 9 horizontal segments. This creates 5*8=40 one by one squares, 4*7=28 two by two squares, 3*6=18 three by three squares, 2*5 = 10 four by four squares, and 1*4=4 five by five squares.
7. The smallest pair is 1 an1, but the smallest nontrivial or distinct pair is 6, 10, found by trial and error.
8. These answers these are unique up to symmetry. I found them mostly by trial and error or a systematic search. One thing that helps is noting which number goes in the center. For parts a and b, there is only one number that can be adjacent to six others. Part c I found only exhaustive search. I would love to see a more elegant way to solve this.

a)



b)



c)